

Class 10 Mathematics — Master Question Solutions

Detailed step-by-step solutions for all questions from Board / SA-1 Examination Papers

SET 1: Model Question Paper (Questions 13 to 18)

Question 13

Show that $\sqrt{5} + \sqrt{7}$ is irrational.

SOLUTION:

Let us assume to the contrary that $\sqrt{5} + \sqrt{7}$ is a rational number.

Then, there exist co-prime integers a and b ($b \neq 0$) such that:

$$\sqrt{5} + \sqrt{7} = a / b$$

Rearranging the equation:

$$\sqrt{7} = (a / b) - \sqrt{5}$$

Squaring both sides:

$$(\sqrt{7})^2 = [(a / b) - \sqrt{5}]^2$$

$$7 = (a^2 / b^2) - 2(a / b)\sqrt{5} + 5$$

$$7 - 5 - (a^2 / b^2) = -2(a / b)\sqrt{5}$$

$$2 - (a^2 / b^2) = -2(a / b)\sqrt{5}$$

$$(2b^2 - a^2) / b^2 = -2(a / b)\sqrt{5}$$

$$(a^2 - 2b^2) / b^2 = 2(a / b)\sqrt{5}$$

Solving for $\sqrt{5}$:

$$\sqrt{5} = (a^2 - 2b^2) / (2ab)$$

Since a and b are integers and $a, b \neq 0$, the RHS $(a^2 - 2b^2) / (2ab)$ is a rational number. This implies that $\sqrt{5}$ must also be rational.

However, this contradicts the known fact that $\sqrt{5}$ is irrational. This contradiction arose due to our incorrect assumption that $\sqrt{5} + \sqrt{7}$ is rational.

Hence, $\sqrt{5} + \sqrt{7}$ is an irrational number. (Proved)

Question 14

$A = \{x : x \text{ is a factor of } 36\},$

$B = \{x : x \text{ is a multiple of } 3, x < 20\}.$

Then find: (i) $A \cup B$, (ii) $A \cap B$, (iii) $A - B$, (iv) $B - A$.

SOLUTION:

Step 1: Write sets A and B in roster form:

- Factors of 36: $A = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$
- Multiples of 3 strictly less than 20: $B = \{3, 6, 9, 12, 15, 18\}$

Step 2: Operations on sets:

1. **Union ($A \cup B$):** All elements in A , B , or both:
 $A \cup B = \{1, 2, 3, 4, 6, 9, 12, 15, 18, 36\}$
2. **Intersection ($A \cap B$):** Elements common to both A and B :
 $A \cap B = \{3, 6, 9, 12, 18\}$
3. **Set Difference ($A - B$):** Elements in A that are NOT in B :
 $A - B = \{1, 2, 4, 36\}$
4. **Set Difference ($B - A$):** Elements in B that are NOT in A :
 $B - A = \{15\}$

Answers:

- $A \cup B = \{1, 2, 3, 4, 6, 9, 12, 15, 18, 36\}$
- $A \cap B = \{3, 6, 9, 12, 18\}$
- $A - B = \{1, 2, 4, 36\}$
- $B - A = \{15\}$

Question 15

Draw the graph of the polynomial $p(x) = x^2 + 2x - 3$ and find the zeroes.

SOLUTION:

Let $y = p(x) = x^2 + 2x - 3$. We construct a table of values:

x	-4	-3	-2	-1	0	1	2
x^2	16	9	4	1	0	1	4
$2x$	-8	-6	-4	-2	0	2	4
-3	-3	-3	-3	-3	-3	-3	-3
y	5	0	-3	-4	-3	0	5
(x, y)	(-4, 5)	(-3, 0)	(-2, -3)	(-1, -4)	(0, -3)	(1, 0)	(2, 5)

Graph Plotting Instructions:

- Plot the coordinates: $(-4, 5)$, $(-3, 0)$, $(-2, -3)$, $(-1, -4)$, $(0, -3)$, $(1, 0)$, $(2, 5)$.
- Join these points smoothly with a free-hand curve to form an upward-opening parabola.
- Vertex of the parabola is at $(-1, -4)$.

Finding the Zeroes:

The zeroes of the polynomial are the x -coordinates of the points where the graph intersects the x -axis ($y = 0$). From the graph and table, the parabola intersects the x -axis at $(-3, 0)$ and $(1, 0)$.

The zeroes of the polynomial $p(x) = x^2 + 2x - 3$ are $x = -3$ and $x = 1$.

Question 16

Construct a triangle of sides 4 cm, 5 cm and 6 cm. Then construct a triangle similar to it whose sides are $\frac{5}{3}$ of corresponding sides.

STEPS OF CONSTRUCTION:**1. Construct original $\triangle ABC$:**

- Draw a base line segment $BC = 6 \text{ cm}$.
- With B as center and radius 5 cm, draw an arc.
- With C as center and radius 4 cm, draw another arc intersecting the previous arc at point A .
- Join AB and AC . Thus, $\triangle ABC$ is formed.

2. Construct ray and mark division points:

- Draw a ray BX making an acute angle with line segment BC on the side opposite to vertex A .
- Since the scale factor is $\frac{5}{3}$ (greater than 1), mark 5 equidistant points on ray BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.

3. Construct similar triangle:

- Join B_3 (the denominator 3) to C .
- From point B_5 , draw a line parallel to B_3C (by constructing an equal angle $\angle BB_5C' = \angle BB_3C$) intersecting the extended line segment BC at point C' .
- From point C' , draw a line parallel to CA intersecting the extended line segment BA at point A' .

Conclusion: $\triangle A'BC'$ is the required enlarged triangle similar to $\triangle ABC$ such that each side is $\frac{5}{3}$ of the corresponding side of $\triangle ABC$.

Question 17

The angle of elevation of the top of a building from foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60° . If the tower is 30 metres high, find the height of the building.

SOLUTION:

Let AB be the tower of height $h_1 = 30 \text{ m}$ and CD be the building of height h_2 .

Let the distance between their bases on level ground be $BD = d$.

1. In right-angled triangle $\triangle ABD$ (Tower):

$$\tan 60^\circ = AB / BD = 30 / d$$

$$\sqrt{3} = 30 / d \Rightarrow d = 30 / \sqrt{3} = 10\sqrt{3} \text{ m}$$

2. In right-angled triangle $\triangle CDB$ (Building):

$$\tan 30^\circ = CD / BD = h_2 / d$$

$$1 / \sqrt{3} = h_2 / (10\sqrt{3})$$

$$h_2 = (10\sqrt{3}) / \sqrt{3} = 10 \text{ m}$$

Height of the building is 10 metres.

Question 18

Find the mode of the following data:

Class Interval	40–45	45–50	50–55	55–60	60–65	65–70	70–75	75–80
Frequency	10	6	8	7	12	9	5	3

SOLUTION:

From the table, the maximum frequency is **12**, corresponding to the class interval **60–65**.

- Modal class = **60–65**
- Lower limit of modal class (l) = **60**
- Frequency of modal class (f_1) = **12**
- Frequency of preceding class (f_0) = **7**
- Frequency of succeeding class (f_2) = **9**
- Class size (h) = **$65 - 60 = 5$**

Formula: $Mode = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h$

Substituting the values:

$$Mode = 60 + \left[\frac{12 - 7}{2(12) - 7 - 9} \right] \times 5$$

$$Mode = 60 + \left[\frac{5}{24 - 16} \right] \times 5$$

$$Mode = 60 + \left[\frac{5}{8} \right] \times 5 = 60 + 25 / 8 = 60 + 3.125 = 63.125$$

Mode of the given data is 63.125.

SET 2: Model Question Paper (Questions 25 to 28)

Question 25

If $\sec \theta + \tan \theta = p$, then prove that $\sin \theta = (p^2 - 1) / (p^2 + 1)$.

PROOF:

Given: $\sec \theta + \tan \theta = p$ --- (Equation 1)

We know the trigonometric identity: $\sec^2 \theta - \tan^2 \theta = 1$

Factoring as difference of squares:

$$(\sec \theta - \tan \theta)(\sec \theta + \tan \theta) = 1$$

$$(\sec \theta - \tan \theta) \cdot p = 1 \Rightarrow \sec \theta - \tan \theta = 1/p \text{ --- (Equation 2)}$$

Adding Equation (1) and Equation (2):

$$2 \sec \theta = p + 1/p = (p^2 + 1)/p \Rightarrow \sec \theta = (p^2 + 1)/(2p)$$

$$\text{Therefore, } \cos \theta = 1/\sec \theta = (2p)/(p^2 + 1)$$

Subtracting Equation (2) from Equation (1):

$$2 \tan \theta = p - 1/p = (p^2 - 1)/p \Rightarrow \tan \theta = (p^2 - 1)/(2p)$$

Now, using the relationship $\sin \theta = \tan \theta \times \cos \theta$ (or $\sin \theta = \tan \theta / \sec \theta$):

$$\sin \theta = [(p^2 - 1)/(2p)] / [(p^2 + 1)/(2p)] = (p^2 - 1)/(p^2 + 1)$$

Hence, $\sin \theta = (p^2 - 1)/(p^2 + 1)$. (Hence Proved)

Question 26

Find the median of the following data:

Class Intervals	0–10	10–20	20–30	30–40	40–50	50–60
Frequency	8	15	19	25	20	13

SOLUTION:

We create the cumulative frequency table:

Class Interval	Frequency (f)	Cumulative Frequency (cf)
0–10	8	8
10–20	15	23
20–30	19	42
30–40 (Median Class)	25	67
40–50	20	87
50–60	13	100
Total	$N = 100$	—

- $N = 100 \Rightarrow N/2 = 50$
- The cumulative frequency just greater than 50 is 67, which belongs to class interval **30–40**.
- Median class = **30–40**
- Lower limit of median class (l) = **30**
- Cumulative frequency preceding median class (cf) = **42**
- Frequency of median class (f) = **25**
- Class width (h) = **10**

Formula: $Median = l + [(N/2 - cf) / f] \times h$

$$Median = 30 + [(50 - 42) / 25] \times 10$$

$$Median = 30 + [8 / 25] \times 10 = 30 + 80 / 25 = 30 + 3.2 = 33.2$$

Median of the given distribution is 33.2.

Question 27

Two poles of heights 6 m and 11 m stand on plain ground. If the distance between feet of the poles is 12 m, find the distance between their tops.

SOLUTION:

Let AB and CD be the two poles standing perpendicular to the ground:

- Height of first pole $AB = 6\text{ m}$
- Height of second pole $CD = 11\text{ m}$
- Distance between their feet $BD = 12\text{ m}$

From the top of the shorter pole A , draw a line segment $AE \perp CD$ meeting CD at point E .

- $ABDE$ forms a rectangle, so $AE = BD = 12\text{ m}$ and $ED = AB = 6\text{ m}$.
- Remaining height $CE = CD - ED = 11 - 6 = 5\text{ m}$.

Now, in right-angled triangle $\triangle AEC$, by Pythagoras Theorem:

$$AC^2 = AE^2 + CE^2$$

$$AC^2 = (12)^2 + (5)^2 = 144 + 25 = 169$$

$$AC = \sqrt{169} = 13\text{ m}$$

The distance between the tops of the poles is 13 metres.

Question 28

Construct a triangle of sides 5 cm, 6 cm and 8 cm, then construct a triangle similar to it whose sides are $\frac{3}{4}$ of the corresponding sides of first triangle.

STEPS OF CONSTRUCTION:**1. Draw the given triangle $\triangle ABC$:**

- Draw a base line segment $BC = 8 \text{ cm}$.
- With center B and radius 5 cm, draw an arc.
- With center C and radius 6 cm, draw another arc intersecting the previous arc at A .
- Join AB and AC . Thus, $\triangle ABC$ is formed.

2. Construct ray and division points:

- Draw a ray BX making an acute angle with BC downwards (opposite side of vertex A).
- Since the scale factor is $\frac{3}{4}$, locate 4 equidistant points B_1, B_2, B_3, B_4 on ray BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

3. Draw the similar triangle:

- Join B_4 (denominator) to C .
- From point B_3 (numerator), draw a line parallel to B_4C intersecting line segment BC at point C' .
- From point C' , draw a line parallel to CA intersecting side BA at point A' .

Conclusion: $\triangle A'BC'$ is the required triangle similar to $\triangle ABC$ with each side equal to $\frac{3}{4}$ of the corresponding sides of $\triangle ABC$.

SET 3: Group - A (Questions 21 to 24)

Question 21

Draw the graph of the polynomial $p(x) = x^2 - 3x + 2$ and find zeroes from the graph.

SOLUTION:

Let $y = p(x) = x^2 - 3x + 2$. Prepare a table of values:

x	-1	0	1	1.5	2	3	4
x^2	1	0	1	2.25	4	9	16
$-3x$	3	0	-3	-4.5	-6	-9	-12
$+2$	2	2	2	2	2	2	2
y	6	2	0	-0.25	0	2	6
(x, y)	(-1, 6)	(0, 2)	(1, 0)	(1.5, -0.25)	(2, 0)	(3, 2)	(4, 6)

Graphing: Plot the coordinates and join them smoothly to form a parabola whose vertex is at $(1.5, -0.25)$.

Zeroes: The curve cuts the x -axis at points $(1, 0)$ and $(2, 0)$.

The zeroes of polynomial $p(x) = x^2 - 3x + 2$ are $x = 1$ and $x = 2$.

Question 22

Show that $\sqrt{5} + \sqrt{7}$ is an irrational number.

PROOF:

Assume on the contrary that $\sqrt{5} + \sqrt{7}$ is rational.

Let $\sqrt{5} + \sqrt{7} = a/b$, where a, b are integers, $b \neq 0$ and co-prime.

$$\sqrt{5} = (a/b) - \sqrt{7}$$

Squaring both sides:

$$5 = (a^2/b^2) - 2(a/b)\sqrt{7} + 7$$

$$2(a/b)\sqrt{7} = 7 - 5 + (a^2/b^2) = 2 + (a^2/b^2) = (a^2 + 2b^2)/b^2$$

$$\sqrt{7} = (a^2 + 2b^2)/(2ab)$$

Since a and b are integers, the RHS is rational, which implies $\sqrt{7}$ is rational. But this contradicts the theorem that $\sqrt{7}$ is irrational.

Therefore, our assumption is false. $\sqrt{5} + \sqrt{7}$ is an irrational number. (Proved)

Question 23

If $A = \{x : x \text{ is a prime factor of } 2310\}$ and $B = \{x : x \text{ is a factor of } 60\}$,
then find: (i) $(A \cup B) - (A \cap B)$ and (ii) $(A - B) \cup (B - A)$.

SOLUTION:

Prime factorization of 2310: $2310 = 2 \times 3 \times 5 \times 7 \times 11$

Set $A = \{2, 3, 5, 7, 11\}$

All factors of 60:

Set $B = \{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$

Common elements:

$A \cap B = \{2, 3, 5\}$

Combined elements:

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 15, 20, 30, 60\}$

Calculations:

$$1. (A \cup B) - (A \cap B) = \{1, 4, 6, 7, 10, 11, 12, 15, 20, 30, 60\}$$

$$2. A - B = \{7, 11\}$$

$$B - A = \{1, 4, 6, 10, 12, 15, 20, 30, 60\}$$

$$(A - B) \cup (B - A) = \{1, 4, 6, 7, 10, 11, 12, 15, 20, 30, 60\}$$

Notice both represent the symmetric difference $A \Delta B$:

$$(A \cup B) - (A \cap B) = (A - B) \cup (B - A) = \{1, 4, 6, 7, 10, 11, 12, 15, 20, 30, 60\}$$

Question 24

Solve the following pair of linear equations:

$$2x + 3y = 9$$

$$3x + 4y = 5$$

SOLUTION:

Given system:

$$2x + 3y = 9 \quad \text{--- (1)}$$

$$3x + 4y = 5 \quad \text{--- (2)}$$

Multiply Equation (1) by 3 and Equation (2) by 2 to equate the coefficients of x:

$$3 \times (2x + 3y = 9) \Rightarrow 6x + 9y = 27 \quad \text{--- (3)}$$

$$2 \times (3x + 4y = 5) \Rightarrow 6x + 8y = 10 \quad \text{--- (4)}$$

Subtracting Equation (4) from Equation (3):

$$(6x - 6x) + (9y - 8y) = 27 - 10$$

$$y = 17$$

Substitute $y = 17$ into Equation (1):

$$2x + 3(17) = 9$$

$$2x + 51 = 9 \Rightarrow 2x = 9 - 51 = -42$$

$$x = -21$$

Solution: $x = -21, y = 17$

SET 4: Paper R-77 (Questions 13 to 18)**Question 13 (R-77)**

Draw the graph of the polynomial $p(x) = x^2 - 2x - 3$ and find its zeroes from the graph.

SOLUTION:

Let $y = p(x) = x^2 - 2x - 3$. Compute coordinates:

x	-2	-1	0	1	2	3	4
x^2	4	1	0	1	4	9	16
$-2x$	4	2	0	-2	-4	-6	-8
-3	-3	-3	-3	-3	-3	-3	-3
y	5	0	-3	-4	-3	0	5
(x, y)	(-2, 5)	(-1, 0)	(0, -3)	(1, -4)	(2, -3)	(3, 0)	(4, 5)

Plot the points and join them with a smooth parabolic curve with vertex at $(1, -4)$.

The graph crosses the x -axis at points $(-1, 0)$ and $(3, 0)$.

The zeroes from the graph are $x = -1$ and $x = 3$.

Question 14 (R-77)

Find the mean of the following data by step deviation method:

Class Interval	1–10	11–20	21–30	31–40	41–50	51–60
Frequency	4	11	13	19	8	5

SOLUTION:

Class width $h = 10$. Let assumed mean $A = 35.5$. Let $u_i = (x_i - A) / h$.

Class Interval	Mid-value (x_i)	Frequency (f_i)	$u_i = (x_i - 35.5)/10$	$f_i u_i$
1–10	5.5	4	–3	–12
11–20	15.5	11	–2	–22
21–30	25.5	13	–1	–13
31–40	35.5 (A)	19	0	0
41–50	45.5	8	1	8
51–60	55.5	5	2	10
Total		$\Sigma f_i = 60$	—	$\Sigma f_i u_i = -39 + 18 = -21$

Formula: $Mean(\bar{x}) = A + [(\Sigma f_i u_i) / (\Sigma f_i)] \times h$

$$\bar{x} = 35.5 + [(-21) / 60] \times 10$$

$$\bar{x} = 35.5 - 21 / 6 = 35.5 - 3.5 = 32$$

Mean by Step Deviation Method = 32.

Question 15 (R-77)

Prove that $5\sqrt{2} - \sqrt{3}$ is an irrational number.

PROOF:

Let us assume that $5\sqrt{2} - \sqrt{3}$ is a rational number equal to $r = a/b$ ($b \neq 0$, $a, b \in \mathbb{Z}$).

$$5\sqrt{2} - r = \sqrt{3}$$

Squaring both sides:

$$(5\sqrt{2} - r)^2 = (\sqrt{3})^2$$

$$25(2) - 10r\sqrt{2} + r^2 = 3$$

$$50 + r^2 - 3 = 10r\sqrt{2}$$

$$47 + r^2 = 10r\sqrt{2}$$

$$\sqrt{2} = (r^2 + 47) / (10r) = [(a/b)^2 + 47] / [10(a/b)] = (a^2 + 47b^2) / (10ab)$$

Since a and b are integers, the expression $(a^2 + 47b^2) / (10ab)$ is rational. Thus, $\sqrt{2}$ must be rational, which contradicts the established theorem that $\sqrt{2}$ is irrational.

Therefore, $5\sqrt{2} - \sqrt{3}$ is irrational. (Proved)

Question 16 (R-77)

7 women and 2 men can together finish a work in 4 days. 3 women and 4 men can finish it in 3 days. Find the time to be taken by one woman alone and one man alone to finish the work.

SOLUTION:

Let time taken by 1 woman alone to complete work be x days.

Let time taken by 1 man alone to complete work be y days.

Then 1 woman's 1-day work = $1/x$, and 1 man's 1-day work = $1/y$.

From given conditions:

$$7/x + 2/y = 1/4 \quad \text{--- (1)}$$

$$3/x + 4/y = 1/3 \quad \text{--- (2)}$$

Let $u = 1/x$ and $v = 1/y$:

$$7u + 2v = 1/4 \Rightarrow \text{Multiply by 2: } 14u + 4v = 1/2 \quad \text{--- (3)}$$

$$3u + 4v = 1/3 \quad \text{--- (4)}$$

Subtracting (4) from (3):

$$11u = 1/2 - 1/3 = 1/6 \Rightarrow u = 1/66$$

Thus, $x = 1/u = 66$ days.

Substitute $u = 1/66$ into equation (4):

$$3(1/66) + 4v = 1/3 \Rightarrow 1/22 + 4v = 1/3$$

$$4v = 1/3 - 1/22 = (22 - 3) / 66 = 19 / 66$$

$$v = 19 / (66 \times 4) = 19 / 264 \Rightarrow y = 264 / 19 = 13 \frac{17}{19} \approx 13.89 \text{ days.}$$

- Time taken by 1 woman alone = 66 days.
- Time taken by 1 man alone = $264/19$ days (or $13 \frac{17}{19}$ days).

Question 17 (R-77)

If $\tan \theta = 15/8$, then find $[15 \cot \theta + 17 \sin \theta] / [8 \tan \theta + 16 \sec \theta]$.

SOLUTION:

Given $\tan \theta = 15/8 = \text{ext}\{\text{Opposite}\} / \text{ext}\{\text{Adjacent}\}$.

By Pythagoras theorem, $\text{ext}\{\text{Hypotenuse}\} = \sqrt{(15^2 + 8^2)} = \sqrt{(225 + 64)} = \sqrt{289} = 17$.

Therefore:

- $\cot \theta = 1 / \tan \theta = 8 / 15$
- $\sin \theta = \text{ext}\{\text{Opposite}\} / \text{ext}\{\text{Hypotenuse}\} = 15 / 17$
- $\sec \theta = \text{ext}\{\text{Hypotenuse}\} / \text{ext}\{\text{Adjacent}\} = 17 / 8$

Evaluate Numerator:

$$15 \cot \theta + 17 \sin \theta = 15(8/15) + 17(15/17) = 8 + 15 = 23$$

Evaluate Denominator:

$$8 \tan \theta + 16 \sec \theta = 8(15/8) + 16(17/8) = 15 + 2(17) = 15 + 34 = 49$$

Value = 23 / 49.

Question 18 (R-77)

Construct a triangle of sides 5 cm, 6 cm and 7 cm, then construct a triangle similar to it, whose sides are $4/3$ times the corresponding sides of the first triangle.

STEPS OF CONSTRUCTION:

1. Draw base line segment $BC = 7 \text{ cm}$.
2. With center B and radius 5 cm, draw an arc. With center C and radius 6 cm, draw an arc intersecting at point A . Join AB and AC to complete $\triangle ABC$.
3. Draw an acute angle ray BX below BC . Mark 4 equidistant points B_1, B_2, B_3, B_4 on BX .
4. Join B_3 to C .
5. From B_4 , draw a line parallel to B_3C to meet extended line BC at C' .
6. From C' , draw a line parallel to CA to meet extended line BA at A' .

$\triangle A'BC'$ is the required similar triangle with side ratio $4/3$.

SET 5: Paper D-115 (Questions 13 to 18)**Question 13 (D-115)**

Prove that $2\sqrt{3} - \sqrt{5}$ is an irrational number.

PROOF:

Assume to the contrary that $2\sqrt{3} - \sqrt{5} = r$, where r is a rational number ($r = a/b$, $b \neq 0$).

$$2\sqrt{3} - r = \sqrt{5}$$

Squaring both sides:

$$(2\sqrt{3} - r)^2 = (\sqrt{5})^2$$

$$4(3) - 4r\sqrt{3} + r^2 = 5$$

$$12 + r^2 - 5 = 4r\sqrt{3}$$

$$7 + r^2 = 4r\sqrt{3}$$

$$\sqrt{3} = (r^2 + 7) / (4r) = [(a/b)^2 + 7] / [4(a/b)] = (a^2 + 7b^2) / (4ab)$$

Since a and b are integers, the RHS is rational, which implies $\sqrt{3}$ is rational. This contradicts the fact that $\sqrt{3}$ is irrational.

Hence, $2\sqrt{3} - \sqrt{5}$ is an irrational number. (Proved)

Question 14 (D-115)

Draw an equilateral triangle of side 3.9 cm. Construct a similar triangle to it with each side is $4/3$ of the corresponding side.

STEPS OF CONSTRUCTION:

1. Draw a line segment $BC = 3.9$ cm.
2. With radius 3.9 cm, draw arcs from B and C intersecting at A . Join AB and AC to form equilateral $\triangle ABC$.
3. Draw ray BX making an acute angle with BC downwards, and mark 4 points B_1, B_2, B_3, B_4 at equal distances.
4. Join B_3 to C .
5. From B_4 , draw a line parallel to B_3C intersecting extended line BC at C' .
6. From C' , draw a line parallel to CA intersecting extended line BA at A' .

$\triangle A'BC'$ is the required equilateral triangle each of whose sides is $4/3 \times 3.9 = 5.2$ cm.

Question 15 (D-115)

Find the median of the following distribution:

Class Interval	40–50	50–60	60–70	70–80	80–90	90–100
Frequency	10	25	28	12	10	15

SOLUTION:

Class Interval	Frequency (f)	Cumulative Frequency (cf)
40–50	10	10
50–60	25	35
60–70 (Median Class)	28	63
70–80	12	75
80–90	10	85
90–100	15	100
Total	$N = 100$	—

- $N/2 = 50$. Class whose cf is just greater than 50 is **60–70**.
- $l = 60$, $cf = 35$, $f = 28$, $h = 10$.

$$\text{Median} = l + [(N/2 - cf) / f] \times h = 60 + [(50 - 35) / 28] \times 10$$

$$\text{Median} = 60 + (15 / 28) \times 10 = 60 + 150 / 28 = 60 + 5.357 = 65.36$$

Median = 65.36.

Question 16 (D-115)

Solve the following pair of equations by reducing them to a pair of linear equations:

$$4/(2x - y) + 7/(2x + y) = 5$$

$$9/(2x - y) - 14/(2x + y) = 7$$

SOLUTION:

Let $u = 1 / (2x - y)$ and $v = 1 / (2x + y)$.

The equations become:

$$4u + 7v = 5 \quad \text{--- (1)}$$

$$9u - 14v = 7 \quad \text{--- (2)}$$

Multiply Equation (1) by 2:

$$8u + 14v = 10 \quad \text{--- (3)}$$

Add Equation (2) and Equation (3):

$$(9u + 8u) + (-14v + 14v) = 7 + 10$$

$$17u = 17 \Rightarrow u = 1$$

Substitute $u = 1$ into (1):

$$4(1) + 7v = 5 \Rightarrow 7v = 1 \Rightarrow v = 1/7$$

Now, substitute back u and v :

$$1 / (2x - y) = 1 \Rightarrow 2x - y = 1 \quad \text{--- (4)}$$

$$1 / (2x + y) = 1/7 \Rightarrow 2x + y = 7 \quad \text{--- (5)}$$

Adding (4) and (5):

$$4x = 8 \Rightarrow x = 2$$

Subtracting (4) from (5):

$$2y = 6 \Rightarrow y = 3$$

Solution: $x = 2, y = 3$.

Question 17 (D-115)

Draw the graph of the quadratic polynomial $x^2 - 2x - 3$ and find the zeroes from the graph.

SOLUTION:

Refer to Question 13 of Paper R-77 above. Setting $y = x^2 - 2x - 3$ gives vertex at $(1, -4)$, with coordinates $(-2, 5)$, $(-1, 0)$, $(0, -3)$, $(1, -4)$, $(2, -3)$, $(3, 0)$, $(4, 5)$.

The parabola crosses the x -axis at $x = -1$ and $x = 3$.

Zeroes from the graph are $x = -1$ and $x = 3$.

Question 18 (D-115)

From the top of tower of 120 m high, a man observes two cars on either side of the tower and in the straight line with the base of the tower with angles of depression as 60° and 45° . Find the distance between the cars.

SOLUTION:

Let AB be the tower of height $h = 120$ m, where B is the base of the tower.

Let the two cars be located at points C and D on opposite sides of the tower along the same horizontal line passing through B .

- Angle of depression to car $C = 60^\circ \Rightarrow \angle ACB = 60^\circ$
- Angle of depression to car $D = 45^\circ \Rightarrow \angle ADB = 45^\circ$

1. In right triangle ΔABC :

$$\tan 60^\circ = AB / BC = 120 / BC$$

$$\sqrt{3} = 120 / BC \Rightarrow BC = 120 / \sqrt{3} = (120\sqrt{3}) / 3 = 40\sqrt{3} \text{ m} \approx 40 \times 1.732 = 69.28 \text{ m}$$

2. In right triangle ΔABD :

$$\tan 45^\circ = AB / BD = 120 / BD$$

$$1 = 120 / BD \Rightarrow BD = 120 \text{ m}$$

Total distance between the two cars (CD):

$$CD = BC + BD = 40\sqrt{3} + 120 = 40(\sqrt{3} + 3) \text{ m}$$

$$CD \approx 69.28 + 120 = 189.28 \text{ m}$$

The distance between the two cars is $40(\sqrt{3} + 3)$ m (or approximately 189.28 metres).

SET 6: Examination Paper — Section - III (Questions 13 to 18)

Note: (i) Answer any Four of the following questions. | (ii) Each question carries 6 marks. [4 × 6 = 24 Marks]

Question 13

Prove that $2 + 3\sqrt{5}$ is an irrational number.

PROOF:

Let us assume to the contrary that $2 + 3\sqrt{5}$ is a rational number.

Then there exist co-prime integers a and b ($b \neq 0$) such that:

$$2 + 3\sqrt{5} = \frac{a}{b}$$

Rearranging the equation to isolate the term containing the radical:

$$3\sqrt{5} = \frac{a}{b} - 2$$

$$3\sqrt{5} = \frac{(a - 2b)}{b}$$

Dividing both sides by 3:

$$\sqrt{5} = \frac{(a - 2b)}{3b}$$

Reasoning & Deduction:

- Since a and b are integers and $b \neq 0$, $(a - 2b)$ and $3b$ are also integers.
- Therefore, the expression $\frac{(a - 2b)}{3b}$ is a rational number.
- This implies that $\sqrt{5}$ must also be a rational number.

However, this contradicts the known fundamental theorem that $\sqrt{5}$ is an irrational number.

This contradiction has arisen due to our incorrect assumption that $2 + 3\sqrt{5}$ is rational.

Hence, $2 + 3\sqrt{5}$ is an irrational number. (Hence Proved)

Question 14

If $A = \{x : x \in \mathbb{N}, x \text{ is a factor of } 30\}$ and $B = \{x : x \in \mathbb{N}, x \text{ is a multiple of } 5, \text{ and } x < 35\}$, then find: (i) $A \cup B$ (ii) $A \cap B$ (iii) $A - B$ (iv) $B - A$.

SOLUTION:

Step 1: Express sets A and B in roster form:

- Factors of 30 that belong to natural numbers $\mathbb{N}$:

$$A = \{1, 2, 3, 5, 6, 10, 15, 30\}$$

- Multiples of 5 in $\mathbb{N}$ strictly less than 35:

$$B = \{5, 10, 15, 20, 25, 30\}$$

Step 2: Compute set operations:

1. **Union ($A \cup B$):** The set of all distinct elements that are in set A, set B, or both:

$$A \cup B = \{1, 2, 3, 5, 6, 10, 15, 20, 25, 30\}$$

2. **Intersection ($A \cap B$):** The set of elements that are common to both set A and set B:

$$A \cap B = \{5, 10, 15, 30\}$$

3. **Difference ($A - B$):** The set of elements belonging to A but NOT in B:

Remove common elements $\{5, 10, 15, 30\}$ from A:

$$A - B = \{1, 2, 3, 6\}$$

4. **Difference ($B - A$):** The set of elements belonging to B but NOT in A:

Remove common elements $\{5, 10, 15, 30\}$ from B:

$$B - A = \{20, 25\}$$

Final Answers:

(i) $A \cup B = \{1, 2, 3, 5, 6, 10, 15, 20, 25, 30\}$

(ii) $A \cap B = \{5, 10, 15, 30\}$

(iii) $A - B = \{1, 2, 3, 6\}$

(iv) $B - A = \{20, 25\}$

Question 15

Draw the graph of the polynomial $p(x) = x^2 - 2x - 3$ and find the zeroes from it.

SOLUTION:

Let $y = p(x) = x^2 - 2x - 3$. We construct a table of values for different values of x :

x	-2	-1	0	1	2	3	4
x^2	4	1	0	1	4	9	16
$-2x$	4	2	0	-2	-4	-6	-8
-3	-3	-3	-3	-3	-3	-3	-3
y	5	0	-3	-4	-3	0	5
(x, y)	(-2, 5)	(-1, 0)	(0, -3)	(1, -4)	(2, -3)	(3, 0)	(4, 5)

Parabola Characteristics:

- **Shape:** Since $a = 1 > 0$, the graph is a smooth upward-opening parabola.
- **Vertex:** Turning point is at $x = -b/(2a) = -(-2)/(2) = 1 \Rightarrow y = 1 - 2 - 3 = -4$. Vertex = (1, -4).

Graph Plotting Instructions:

1. Take scale on graph sheet: On X-axis: 1 cm = 1 unit; On Y-axis: 1 cm = 1 unit.
2. Plot the coordinates: (-2, 5), (-1, 0), (0, -3), (1, -4), (2, -3), (3, 0), (4, 5).
3. Join these plotted points smoothly by free-hand to obtain the required parabola.

Finding the Zeroes:

The zeroes of the polynomial are the x -coordinates of the points where the parabola intersects the x -axis (i.e. where $y = 0$).

From the graph, the curve cuts the x -axis at (-1, 0) and (3, 0).

Therefore, the zeroes of the polynomial $p(x) = x^2 - 2x - 3$ are $x = -1$ and $x = 3$.

Question 16

Find the median of the following data:

Class Interval	45–65	65–85	85–105	105–125	125–145	145–165	165–185
Frequency	4	7	8	3	12	9	7

SOLUTION:

We construct the cumulative frequency table:

Class Interval	Frequency (f)	Cumulative Frequency (cf)
45 – 65	4	4
65 – 85	7	$4 + 7 = 11$
85 – 105	8	$11 + 8 = 19$
105 – 125	3	$19 + 3 = 22$
125 – 145 (Median Class)	12	$22 + 12 = 34$
145 – 165	9	$34 + 9 = 43$
165 – 185	7	$43 + 7 = 50$
Total	$N = \Sigma f = 50$	—

Here, total frequency $N = 50 \Rightarrow \frac{N}{2} = \frac{50}{2} = 25$.

The cumulative frequency just greater than 25 is **34**, which corresponds to the class interval **125 – 145**.

- **Median Class** = 125 – 145
- Lower limit of median class (l) = **125**
- Cumulative frequency preceding median class (cf) = **22**
- Frequency of median class (f) = **12**
- Class width (h) = $145 - 125 = 20$

Formula: $Median = l + \left[\left(\frac{N}{2} - cf \right) / f \right] \times h$

Substituting the corresponding values into the formula:

$$Median = 125 + \left[(25 - 22) / 12 \right] \times 20$$

$$Median = 125 + \left[3 / 12 \right] \times 20$$

$$Median = 125 + \left[1 / 4 \right] \times 20$$

$$Median = 125 + 5 = 130$$

The median of the given data is 130.

Question 17

Construct a triangle PQR, in which PQ = 5 cm, QR = 6 cm and PR = 7 cm, then construct a triangle similar to it, whose sides are $\frac{3}{5}$ of the corresponding sides of the first triangle.

STEPS OF CONSTRUCTION:

Step 1: Construct the given triangle ΔPQR :

- Draw a straight line segment $QR = 6 \text{ cm}$ as base.
- With point Q as center and radius 5 cm (length of PQ), draw an arc using a compass.
- With point R as center and radius 7 cm (length of PR), draw another arc intersecting the previous arc at point P.
- Join PQ and PR. Thus, the given triangle ΔPQR is constructed.

Step 2: Construct the auxiliary ray and division points:

- Draw a ray QX making an acute angle with the base segment QR on the side opposite to vertex P (downwards).
- Since the given scale factor is $\frac{3}{5} < 1$, identify the greater number between numerator 3 and denominator 5, which is 5.
- Locate 5 equidistant points on ray QX: mark Q_1, Q_2, Q_3, Q_4, Q_5 such that:

$$QQ_1 = Q_1Q_2 = Q_2Q_3 = Q_3Q_4 = Q_4Q_5$$

Step 3: Construct the similar triangle $\Delta P'QR'$:

- Join the 5th point (the denominator 5) Q_5 to vertex R (i.e. draw segment Q_5R).
- From point Q_3 (the numerator 3), draw a line parallel to Q_5R by constructing an angle equal to $\angle QQ_5R$, intersecting the line segment QR at point R' .
- From point R' , draw a straight line parallel to side RP by constructing an angle equal to $\angle QRP$, intersecting the side QP at point P' .

Justification:

Since $Q_3R' \parallel Q_5R$, by Basic Proportionality Theorem in ΔQQ_5R :

$$\frac{QR'}{QR} = \frac{QQ_3}{QQ_5} = \frac{3}{5}.$$

Similarly, since $P'R' \parallel PR$, $\Delta P'QR' \sim \Delta PQR$, which implies:

$$\frac{P'Q}{PQ} = \frac{QR'}{QR} = \frac{P'R'}{PR} = \frac{3}{5}.$$

Conclusion: $\Delta P'QR'$ is the required smaller similar triangle whose sides are $\frac{3}{5}$ of the corresponding sides of ΔPQR .

Question 18

The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60° . If the tower is 60 m high, find the height of the building.

SOLUTION:

Let AB be the tower standing vertically on the ground of height $H = 60 \text{ ext{ m}}$.

Let CD be the building standing vertically on the ground of height h .

Let $BD = d$ be the horizontal distance between the foot of the tower (B) and the foot of the building (D).

Given Angles of Elevation:

- Angle of elevation of the top of tower (A) from foot of building (D): $\angle ADB = 60^\circ$
- Angle of elevation of the top of building (C) from foot of tower (B): $\angle CBD = 30^\circ$

Step 1: In right-angled triangle $\triangle ABD$ (Tower):

$$\begin{aligned}\tan 60^\circ &= \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{AB}{BD} \\ \sqrt{3} &= \frac{60}{d} \\ d &= \frac{60}{\sqrt{3}} = \frac{(60 \times \sqrt{3})}{3} = 20\sqrt{3} \text{ ext{ m}}\end{aligned}$$

Step 2: In right-angled triangle $\triangle CDB$ (Building):

$$\begin{aligned}\tan 30^\circ &= \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{CD}{BD} \\ \frac{1}{\sqrt{3}} &= \frac{h}{d} \\ h &= \frac{d}{\sqrt{3}}\end{aligned}$$

Substituting the value of $d = 20\sqrt{3} \text{ ext{ m}}$ from Step 1:

$$h = \frac{(20\sqrt{3})}{\sqrt{3}} = 20 \text{ ext{ m}}$$

Verification via Direct Ratio:

Dividing the two trigonometric relations: $\frac{\tan 30^\circ}{\tan 60^\circ} = \frac{(h/d)}{(H/d)} = \frac{h}{H}$
 $\frac{(1/\sqrt{3})}{\sqrt{3}} = \frac{1}{3} = \frac{h}{60} \Rightarrow h = \frac{60}{3} = 20 \text{ ext{ m}}.$

The height of the building is 20 metres.